TOPOLOGICAL DERIVATIVE FOR THE INVERSE SCATTERING OF ELASTIC WAVES

Bojan B. Guzina[a] and **Marc Bonnet**[b]

[a]Department of Civil Engineering
University of Minnesota
Minneapolis, MN 55455
guzina@wave.ce.umn.edu

[b]Laboratoire de Mécanique des Solides
École Polytechnique
F-91128 Palaiseau Cedex
France

To establish an alternative analytical framework for the elastic-wave imaging of underground cavities, the focus of this study is an extension of the concept of topological derivative, rooted in elastostatics and shape optimization, to 3D elastodynamics involving semi-infinite and infinite solids. The main result of the proposed boundary integral approach is a formula for the topological derivative $\mathcal{T}(\mathbf{z})$, explicit in terms of the dynamic fundamental solution, obtained by an asymptotic expansion of the misfit-type cost functional with respect to the creation of an infinitesimal hole in an otherwise intact (semi-infinite or infinite) elastic medium. Valid for an arbitrary shape of the infinitesimal cavity, the formula involves the solution to six canonical exterior elastostatic problems, and becomes fully explicit when the vanishing cavity is spherical. A set of numerical results is included to illustrate the potential of topological derivative as a computationally efficient tool for exposing an approximate cavity location via a grid-type exploration of the host domain. For a comprehensive solution to 3D inverse scattering problems involving elastic waves, the proposed approach can be used most effectively as a pre-conditioning tool for more refined, albeit computationally intensive minimization-based imaging algorithms. To the authors' knowledge, an application of topological derivative to inverse scattering problems has not been attempted before; the methodology proposed in this paper could also be extended to acoustic problems.

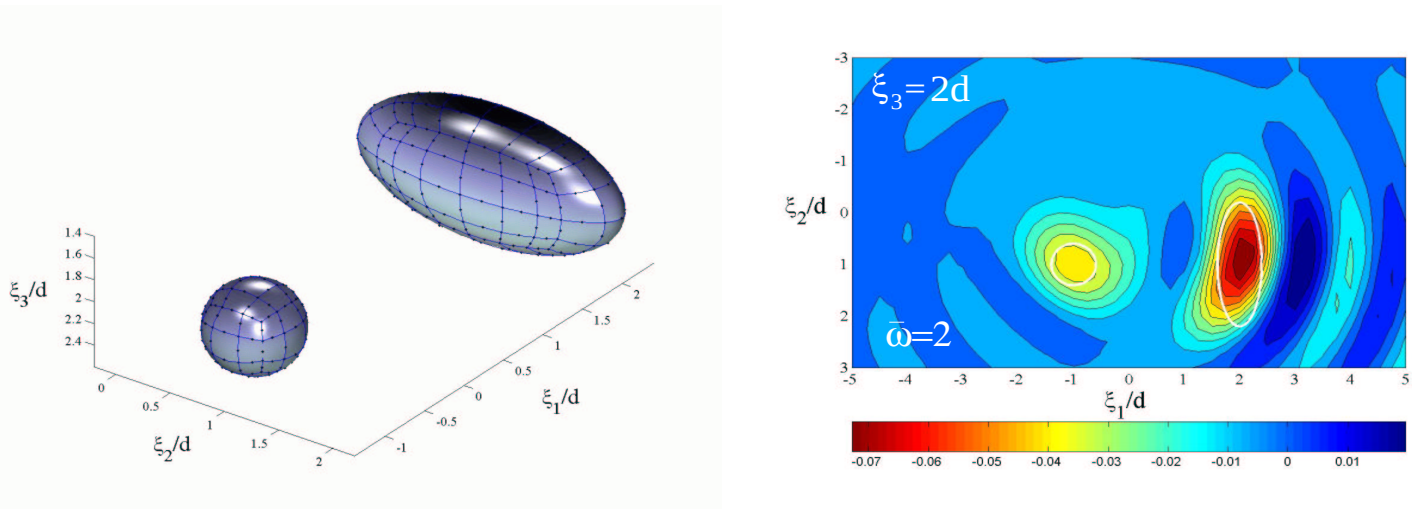

Figure 1: Application of topological derivative ($\mathcal{T}$) to the elastic-wave imaging of a uniform semi-infinite solid ($\xi_3 > 0$): **a)** dual cavity configuration; **b)** distribution of $\mathcal{T}(\mathbf{z})$ in the horizontal plane containing centroids of both cavities.